Simulation of Galactic Dark Matter Interactions with Nuclei in the SuperCDMS Experiment

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Outline

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- SuperCDMS Experiment
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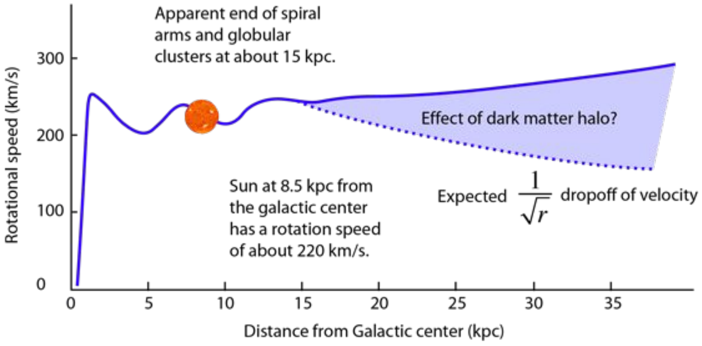
Dark Matter and the Weakly Interacting Massive Particle Hypothesis

Dark Matter - Evidence from the orbital speeds of stars in the Milky Way

- The expected rotational velocity of an object at some radius r from a mass distribution M(r) for a central force is:

$$v_{\text{rot}} = \sqrt{\frac{G_{\text{N}} M(r)}{r}}$$

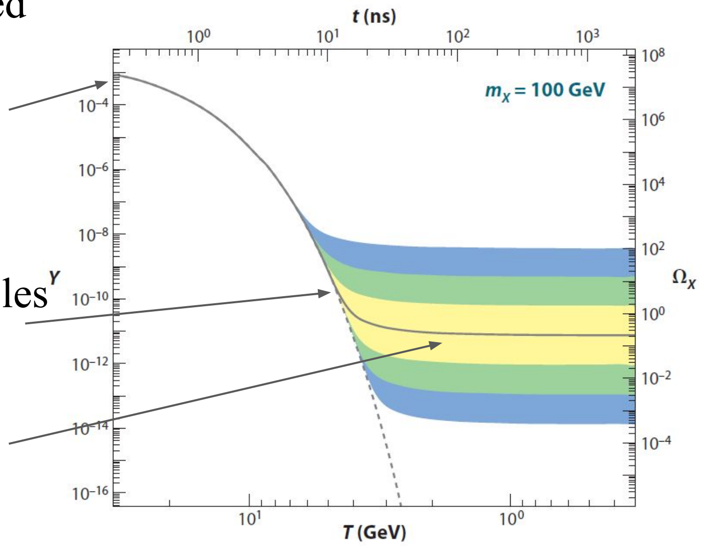
- The observed speeds of stars orbiting the center of a galaxy:
  - Do not fall as $r^{-1/2}$ at high radii
  - Have a velocity distribution that is almost flat as a function of the distance
  - This implies a mass distribution of $\rho_{(r)} \propto r^{-2}$
- This suggests that that there is a (roughly) spherical halo of mass that surrounds the stars within the galaxy. We call this mass Dark Matter as we can't see any light from it



[5],[9]

Dark Matter - Cosmology Prediction

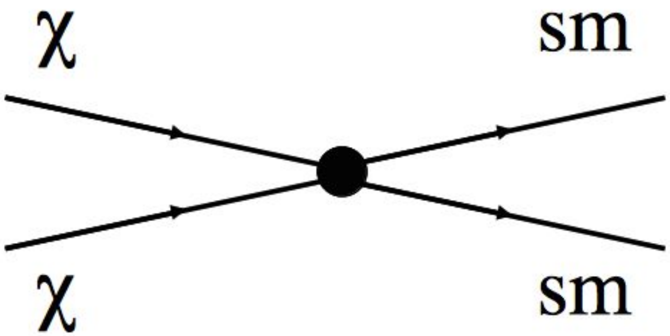
- The standard model of Cosmology suggests that if dark matter was a particle that was created in the early universe and evolved like other particles:
  - Early times: The temperature was so high that the dark matter particle would be in thermal equilibrium with the other particles with constant creation and annihilation
  - Later: Temperature drops so much that dark matter particles cannot be produced anymore, while the annihilation continues
  - Even later: Universe expanded enough that annihilation stops and the number density of the dark matter particles becomes constant
- The number density when annihilation stops, depends on the mass and self-interaction cross section of dark matter particles



Demonstration of freeze-out for particle m_x [6]

Weakly Interacting Massive Particles - WIMP Model Motivation

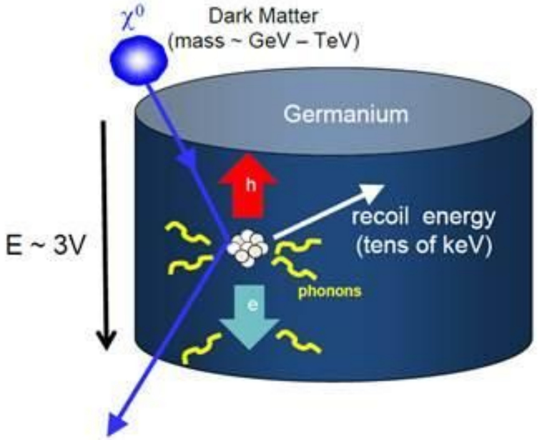
- With some simplifying assumptions we can make predictions about the properties of dark matter:
  - There is only one type of dark matter particle
  - Dark matter is its own antiparticle
- The measured abundance of dark matter from the cosmic background radiation allows for an estimate of the mass/cross section combination. For heavy dark matter particles it suggests that the dark matter particles have the same interaction strength as weakly interacting particles [8]
- We call this type of dark matter, Weakly Interacting Massive Particles (WIMPs)
- If this hypothesis is correct, then dark matter particles should interact weakly with a nucleus at a rate that can observed in an experiment like the Super Cryogenic Dark Matter Search (**SuperCDMS)**

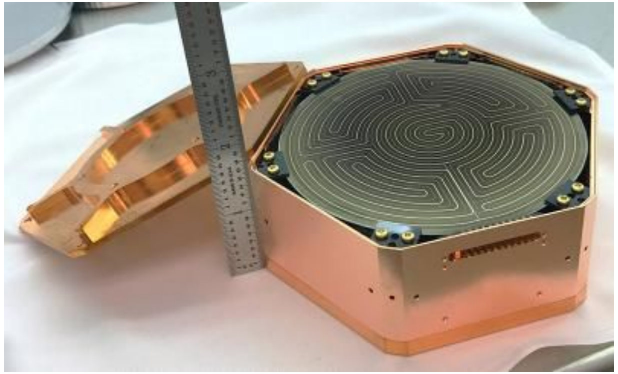


SuperCDMS

SuperCDMS

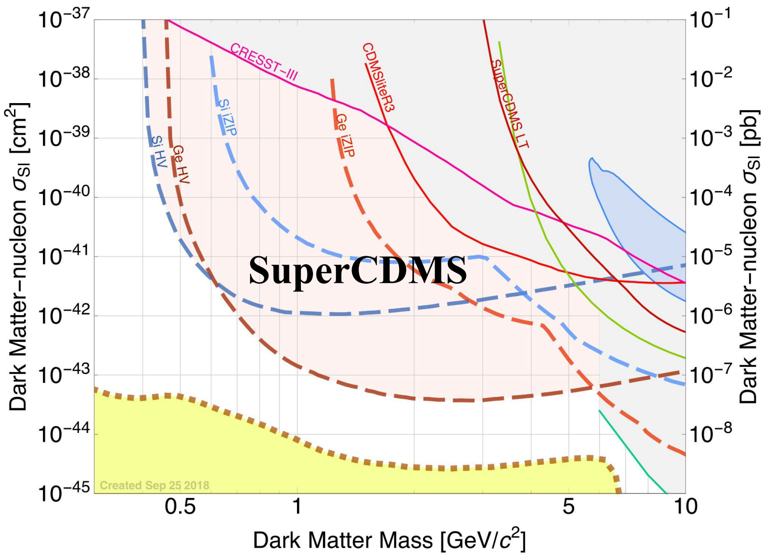
- The prediction is that the sun is moving through the dark matter halo that surrounds the Milky Way so we expect that the Earth is passing through a flux of dark matter
- SuperCDMS is looking for single interactions between an atomic nucleus in the detector and a dark matter particle
    - The experimental observable is the recoil energy of the nucleus
- SuperCDMS detector towers work at 50mK
    - Thermal motion of the nuclei in the detector is negligible





SuperCDMS - Sensitivity

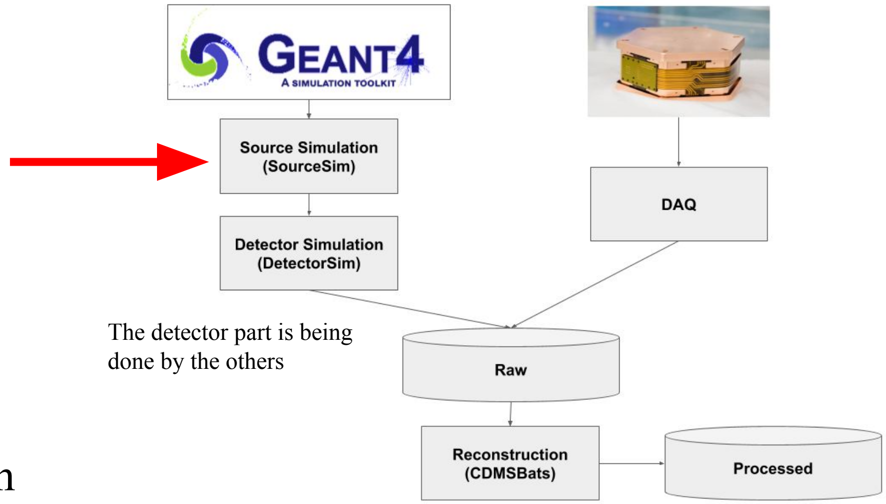
- This picture shows the expected sensitivity (dark matter-nucleon interaction cross section) as a function of the dark matter mass
- SuperCDMS will have 4 types of detectors with two different atomic types between them (Germanium and Silicon):
  - We are sensitive to particles above about 0.5 GeV/c^2
  - SuperCDMS is the most sensitive detector below about 10 GeV/c^2
- Our goal is to have a full simulation for WIMPs within the SuperCDMS sensitivity range



Simulation Goals

Simulation Goals

- We want a reliable signal generator for WIMPs that uses the standard tools to fully simulate and process individual events like the real data
- Need to take into account known physics
    - Galactic physics, 4-momentum WIMP-nucleus kinematics, and nuclear/particle physics
- We will refer to this simulation as WimpSim



Galactic physics, 4-momentum WIMP-nucleus kinematics, and nuclear/particle physics

Overview of the Physics Behind our expected Recoil Energy Distribution

There are three main pieces of physics which affect the expected recoil energy distribution of the nucleus:

1. The kinematics of a classical elastic scattering
2. WIMP velocity distribution (Galactic Modeling)
3. Differential cross section for WIMP-nucleon interactions taken from the nuclear and particle physics of the interaction (taken to be a function of recoil energy)

Recoil Energy Distribution

$$\frac{\mathrm{d}R}{\mathrm{d}E_{\mathrm{r}}} = \underbrace{\frac{N_T m_T}{2 m_\chi \mu_T^2}}_{1} \cdot \underbrace{\left[\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(E_{\mathrm{r}}) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(E_{\mathrm{r}})\right]}_{2} \cdot \underbrace{\rho_0 \int_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} \frac{1}{k} \frac{f(\boldsymbol{v}, \boldsymbol{v}_E)}{v} \, \mathrm{d}^3\boldsymbol{v}}_{3}$$

While it's not how we do the simulation, for simplicity we write the recoil energy distribution as coming from 3 separate parts: (see backup slides for more info about the equation)

1. A scaling part which is a constant factor and is dependent on the different nuclei masses in the detector as well as the WIMP mass
2. Differential Cross Section: Which describes the nuclear and particle physics of the interaction
3. Classical Part: This part has two pieces inside of it:
   a. Galactic Modeling: Which is our WIMP velocity distribution
   b. Classical Scattering: Which is basically the billiard ball problem

M. Pepin, "Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search"

N_T	nuclei per unit mass
m_T	nuclei mass
m_χ	WIMP Mass
μ_T	reduced mass
F_{SI}	spin-independent form factor
F_{SD}	spin-dependent form factor
$f(v,v_E)$	WIMP velocity distribution function
v	Velocity of a WIMP
v_E	Velocity of the Earth
σ_0	total cross section in the zero momentum limit
ρ_0	local mass density of WIMPs
k	Integral normalization factor

Simulation Assumptions

Assumptions - Galactic Modeling

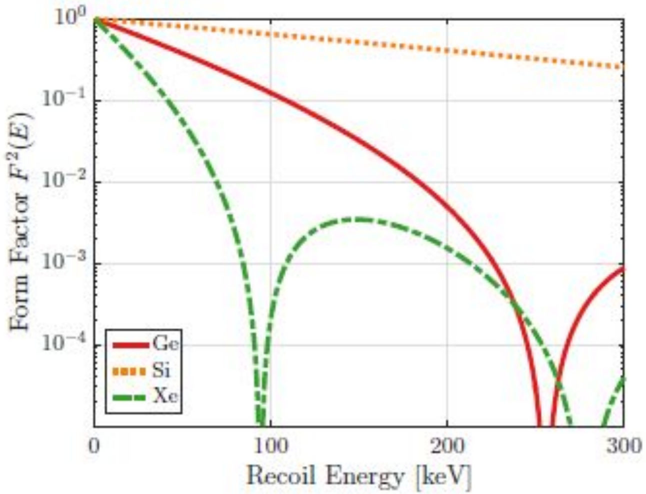
$$\underbrace{\rho_0 \int_{v_{\min}}^{v_{\max}} \frac{1}{k} \frac{f(v, v_E)}{v} \mathrm{d}^3 v}_{\text{Astro}}$$

- In WimpSim we assume the dark matter halo in the Milky Way is an isothermal, isotropic, and non-rotating sphere [1]
- The velocity distribution of WIMPs is modeled as a gas in thermal equilibrium inside the radially symmetric gravitational potential well in the galaxy
- WIMP velocity magnitude distribution assumed to be a Maxwell-Boltzmann distribution with a mean speed of 220 km/s (See appendix B [3])
- Sun's velocity assumed to be equal to 232 km/s and always in the same direction
- Earth's phase around the Sun is uniformly distributed (with a speed constant of 29.79 km/s taking into account the ellipticity of Earth's Orbit and Minor Axis Longitude) (see [3] page 110)
- We assume that there will be no WIMP above the galaxy escape velocity

Differential Cross Section - Helm form factor
Description of modeling

- Since WIMP-nucleon interactions are not well understood, but likely to have both hadronic and nuclear components, we model them with simple assumptions
- Separate the components of the interaction into two terms:
  - Spin independent
  - Spin dependent
- For each term separate into the total cross section and take a differential correction factor:
  - Total cross section term in the zero-momentum limit
  - Differential term modeled as a nuclear form factor which is recoil energy dependent
- For simplicity WimpSim assumes all WIMP-nucleon interactions are spin independent (see backup slides for spin dependent comments)
  - Spin dependent terms are taken to be zero
- We use Helm form factor which is the one most commonly used in the direct detection community[2] and has the potential advantage of one day being measurable

$$\underbrace{\left[\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(E_{\mathrm{r}}) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(E_{\mathrm{r}})\right]}_{\text{Particle and Nuclear}}$$



Assumptions - Classical Scattering

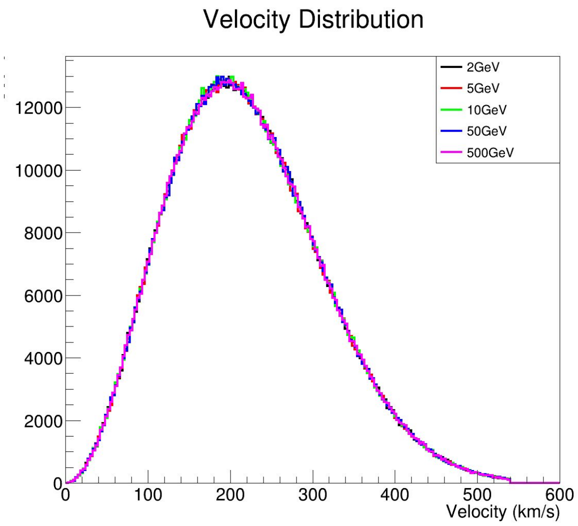
- The collision is done in the rest frame of the Earth
- All atoms are assumed to be at rest for all the different atom types in the crystal (see backup slides for Thermal Motion's effect)
    - Atoms velocities are assumed to be equal to the Earth's velocity
- All collisions are assumed to be elastic
    - Collisions does not change the nucleus's state
    - No proton or neutron ejection, No nuclear excitation, etc.

How WimpSim Works

Converting our assumptions into useable results: Events and Event Weights

How WimpSim Works - Galactic Modeling

- Since we assume an isotropic model, and theoretical predictions indicate a mean velocity of 220 km/s at the location of the Sun, we randomly generate the WIMP velocity in each direction using a Gaussian distribution with a mean at 0 and an RMS of $220/(2\sqrt{(2/\pi)}) = 138$ km/s
- For simplicity, we implemented a cut off at 544 km/s for WIMP velocity which is the escape velocity of the galaxy around the sun [4]



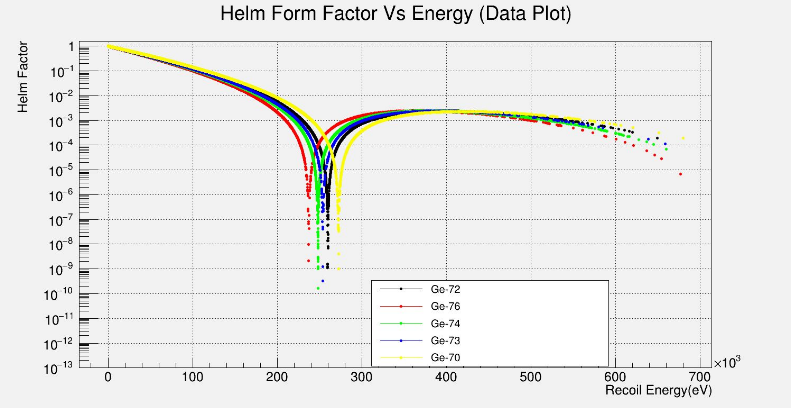
How WimpSim Works - Classical Scattering

- WIMP-nucleon interactions are expected to have a very small cross section that is recoil energy dependent. For technical reasons, and future expansion, we create samples of events where:
  - WimpSim always simulates an event
  - The location of the interaction in the detector is chosen randomly
- The nuclear mass is chosen randomly based on the composition of the detector material
- The WIMP incident angle is chosen from a uniform distribution in the WIMP-nucleon center of mass frame
- Sun velocity vector in WimpSim is constant
- Earth phase is random and we boost WIMP's velocity in Earth's frame
- The outgoing angle is uniformly distributed in the center of the nucleon-WIMP reference frame, and we just transform back to the lab frame

Variable	Unit	How WimpSim calculates it
WIMP mass	eV	Input parameter
WIMP Initial Velocity In Galactic frame	m/s	Taken from Galactic Model
WIMP Initial Velocity Relative to Earth	m/s	Taken from Galactic Model
WIMP Initial Energy	eV	Taken from Galactic Model
WIMP Incident Angles	Radians	Uniform distribution in the WIMP-nucleon center of mass frame
Nucleus Recoil Energy	eV	Classical scattering
WIMP Momentum After Collision	eV	Classical scattering Outgoing angle is uniformly distributed in CM frame
Nucleus Momentum After Collision	eV	Classical scattering Outgoing angle dependent on WIMP's outgoing angle
Helm form factor	Unitless	Calculated from recoil energy

How WimpSim Works - Differential Cross Section Correction Factor

- WimpSim takes the recoil energy in the lab frame, looks up the corresponding spin-independent Form Factor (Helm Form Factor as the model) for that isotope and puts it in the output as a weight for use in making recoil energy distributions
- The plot at the right side shows the Helm Form Factor as a function of recoil energy for different isotopes
  - In the SuperCDMS primary mass range (0.5 GeV/c^2 to 10 GeV/c^2) with the 544 km/s WIMP velocity cut-off, the Helm Form Factor would be close to 1

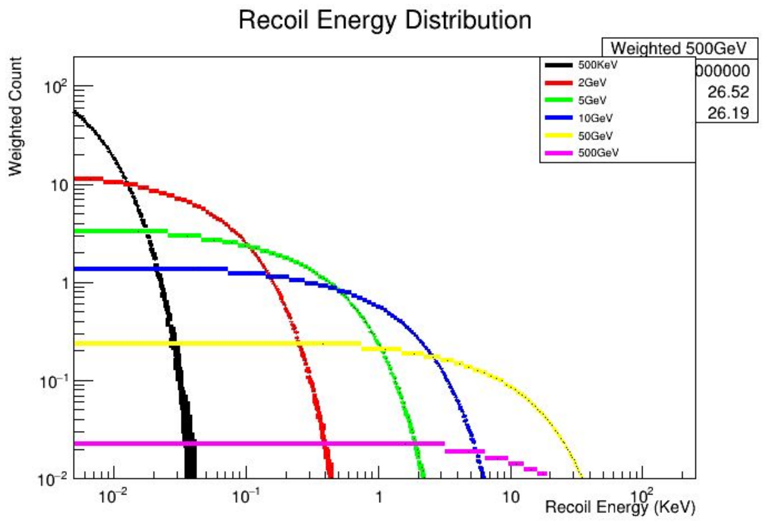


Results

Recoil Energy Distribution

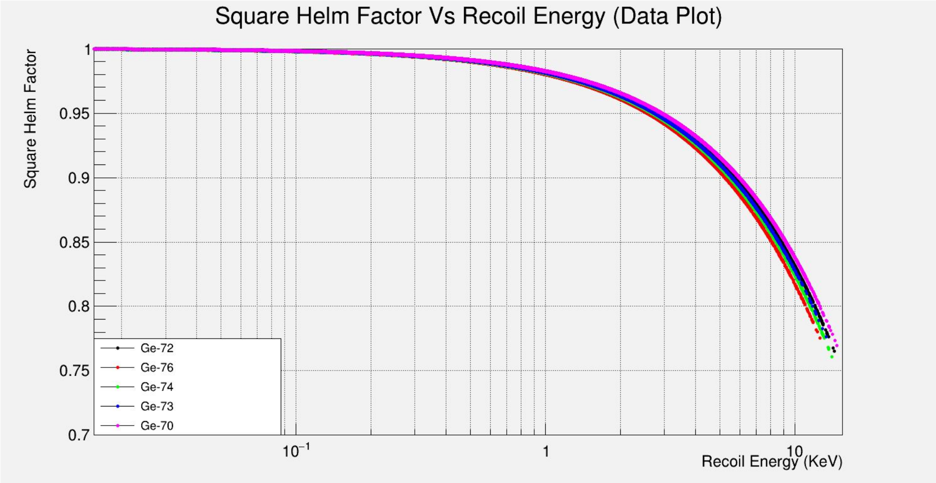
- The plot shows our recoil energy distribution results for a million events

In the next pages we will look at the 10 GeV and 0.5 GeV cases to show how WimpSim works for the SuperCDMS strong sensitivity range and then 500 GeV sample to show the exaggerated effect of Helm Form Factor

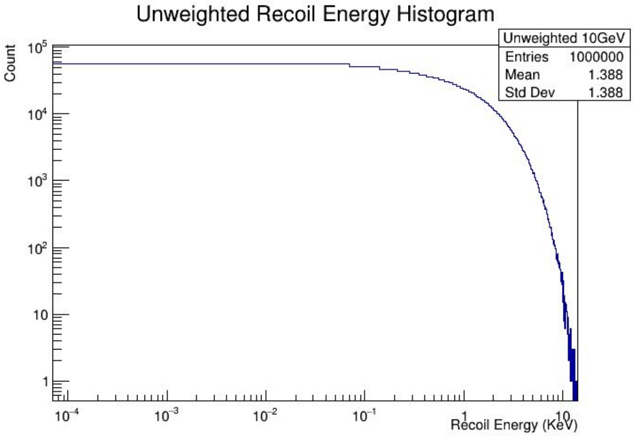


Helm From Factor - 10 GeV sample

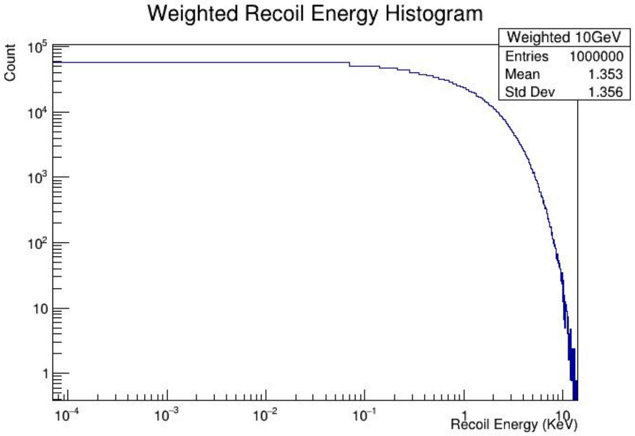
- As expected, Helm form factor is different for different isotopes
- Their difference is not significant

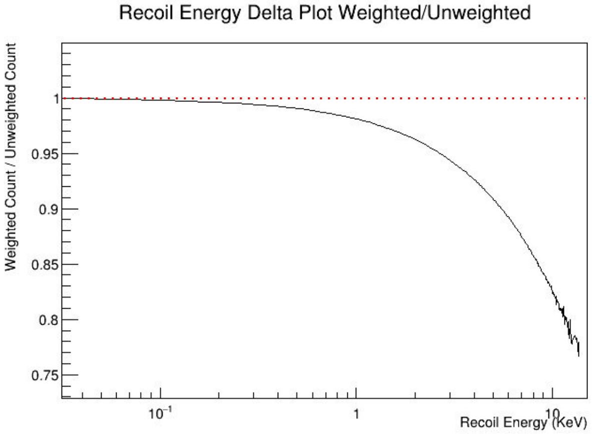


Recoil Energy Distribution - 10 GeV sample



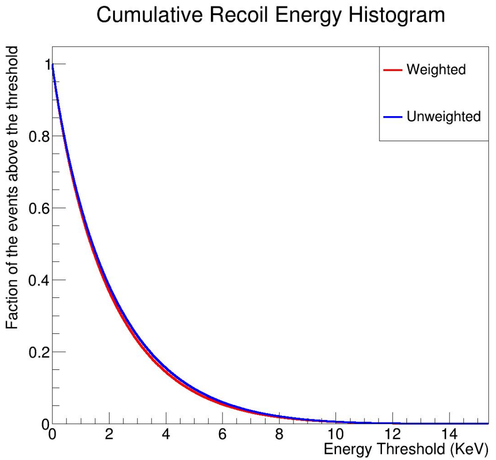
- The left plots show unweighted and weighted recoil energy distributions
- In the 10 GeV case there is not much difference between the two
- The ratio (which is equal to the form factor, averaged over all isotopes) is shown on the bottom right.





Impact of the form factor on Various Threshold Cuts - 10 GeV Sample

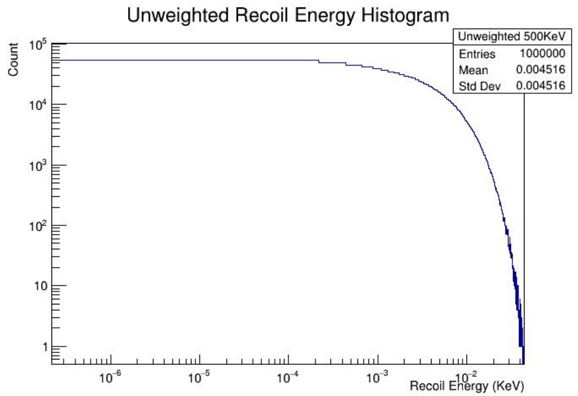
- Typically, an analysis is looking for the fraction of events above an energy threshold.
- The plot on the right shows what faction of the events (y axis) has an energy above a threshold (x axis) for both weighted and unweighted events
- Note that for low mass WIMPs, the effect of Helm form factor only has a small impact on the distribution as most of the events have small recoil energy



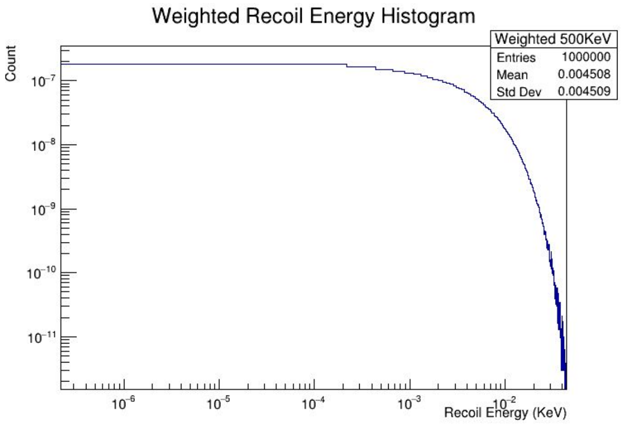
	% Above 60eV	% Above 0.1keV	% Above 0.2keV
Weighted	94.77%	91.57%	84.32%
Unweighted	94.90%	91.78%	84.70%

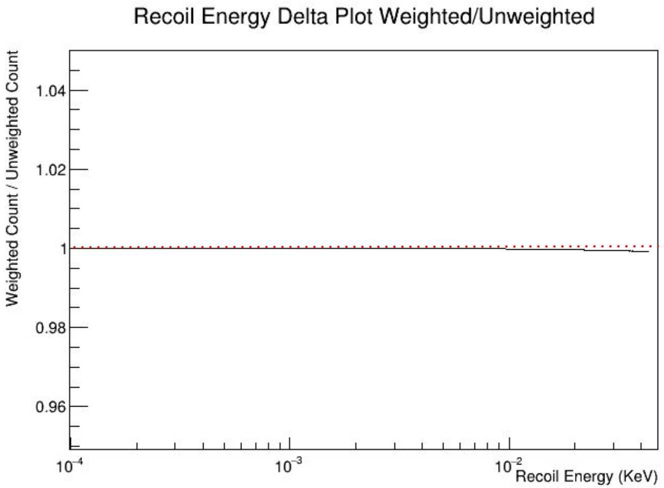
	Threshold that gives 68.2% efficiency	Threshold that gives 95.4% efficiency	Threshold that gives 99.6% efficiency
Weighted	0.476keV	0.053keV	0.004keV
Unweighted	0.489keV	0.054keV	0.004keV

Recoil Energy Distribution - 0.5 GeV sample

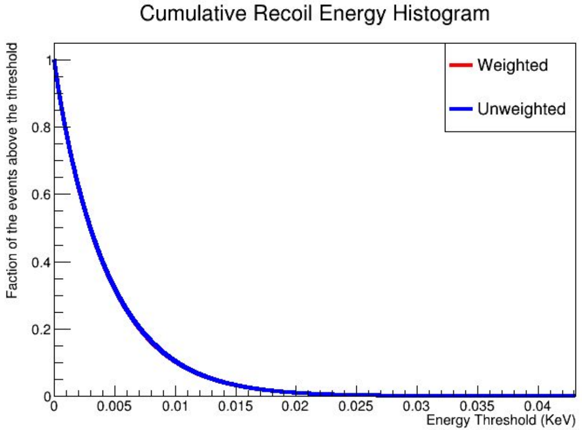


- The overall shape of the recoil energy distribution does not change when the weights are taken into account





Impact of the form factor on Various Threshold Cuts - 0.5 GeV Sample



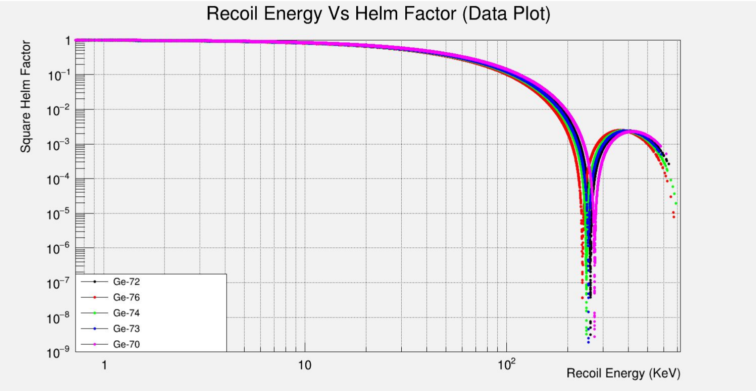
- The difference between the weighted and unweighted samples are extremely small

	% Above 3eV	% Above 5eV	% Above 10eV
Weighted	49.89%	31.99%	10.33%
Unweighted	49.95%	32.05%	10.38%

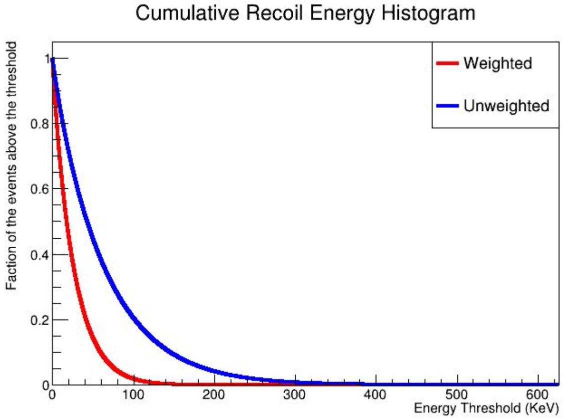
	Threshold that gives 68.2% efficiency	Threshold that gives 95.4% efficiency	Threshold that gives 99.6% efficiency
Weighted	1.607eV	1.787eV	$1.46*10^{-5}$eV
Unweighted	1.610eV	1.790eV	$1.47*10^{-5}$eV

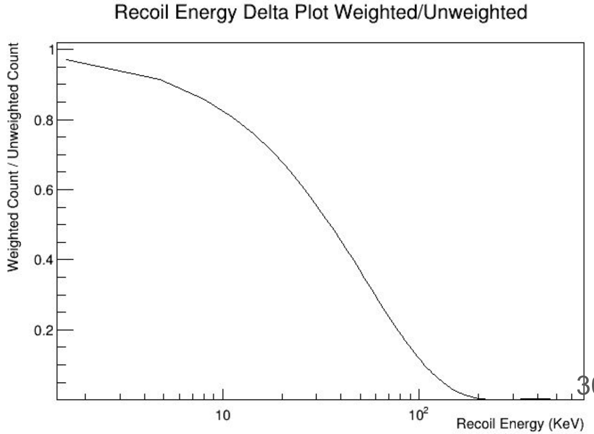
Recoil Energy Distributions at Higher Masses - 500 GeV Sample

- The 500 GeV sample shows the exaggerated effect of Helm Form Factor
- For the higher masses we start to get larger recoil energies, including large enough values to see the dip
- The overall shape of the recoil energy distribution plot changes significantly when the weights are taken into account
- Unlike the low mass WIMPs, the difference between weighted and unweighted Recoil Energy Distribution is big which means the effect of weighting for high mass WIMPs is significant



	Above 68.2% Threshold	Above 95.4% Threshold	Above 99.6% Threshold
Weighted	9.58keV	1.09keV	0.09keV
Unweighted	23.02keV	2.56keV	0.20keV





Potential Future Improvements

- Different form factor calculations (or different Helm form factor constants see [3])
- Spin-dependent WIMP-nucleon interactions
- Inelastic scattering (anything that changes the nucleus' state, for example a proton or neutron gets ejected, or a state gets excited)

Conclusion

- SuperCDMS is searching for dark matter and is sensitive to WIMP masses in the range between 0.5 to 10 GeV/c^2
- We have simulated the detectors response between WIMPs and nuclei for the SuperCDMS experiment
    - WimpSim contains known galactic and nuclear/particle physics
    - It uses reasonable defaults in its modeling, is ready for expansion, and currently provides both individual events and event weights with Helm Form factors for spin-independent interactions
- We are looking forward to using WimpSim to discover WIMPs

References

1. M. Pepin, “Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search”
2. R. H. Helm, “Inelastic and Elastic Scattering of 187-Mev Electrons from Selected Even-Even Nuclei”
3. J.D. Lewin, RF. Smith “Review of mathematics, numerical factors, and corrections dark matter experiments based on elastic nuclear recoil” 87-112
4. T. Piffl, “The RAVE survey: the Galactic escape speed and the mass of the Milky Way"
5. Carroll, Bradley W. and Ostlie, Dale A., “An Introduction to Modern Astrophysics, 2nd Ed.”
6. J. L. Feng, “Dark Matter Candidates from Particle Physics and Methods of Detection”
7. P. A. R. Ade et al. (Planck Collaboration), “Planck 2015 results: XIII. Cosmological parameters"
8. Marc Kamionkowski, “WIMP and Axion Dark Matter”
9. Rohlf, James William, “Modern Physics from a to Z0”
10. E. Corbelli1w, P. Salucci “The extended rotation curve and the dark matter halo of M33”

Backup Slides

How much does your work improve the sensitivity of CDMS to dark matter?

Hypothesis testing requires background and signal data. The difference between different groups setups are in their equipments which cause background data.

Improving sensitivity of an experiment, means increasing the cross section of the Dark Matter-nucleon ($\boldsymbol{\sigma}_{\mathrm{si}}$ and $\boldsymbol{\sigma}_{\mathrm{SD}}$) . These cross sections are completely dependent on our equipments and geometry and methods of data acquisition.

WimpSim's is a signal generator and is "producing" the sensitivity plots (by producing the $\boldsymbol{\sigma}_{\mathrm{si}}$ and $\boldsymbol{\sigma}_{\mathrm{SD}}$ versus WIMP's mass plots). Therefore, it is not increasing the sensitivity of CDMS. WimpSim and it's improvements, gives us a better "estimate" of the sensitivity of CDMS.

WimpSim V1

- WimpSim V1 updated by Nate Herbert at 8/29/2019
  - https://confluence.slac.stanford.edu/download/attachments/258184131/Simulations%20Talk%20-%20Nate%20Herbert%5B70023%5D.pdf?version=1&modificationDate=1565035140000&api=v2
- Since then we found five problems which are now fixed
  - Recoil calculations were wrong, and scattering angles were all in the same directions, and momentum was not conserved. (Nucleus momentum after the hit was getting calculated by simply dividing the magnitude into 3 equal parts)
  - Helm form factor calculations were wrong (There was a variable declaration issue that were causing that all the sq terms (a Helm form factor term) to have zero effect)
  - WIMP velocity distribution had a bug that were resulting into a Maxwell-Boltzmann distribution with a mean at 330 km/s instead of 220 km/s
  - Data members were being initialized with a non-zero value (which could cause issues in other simulations)
  - Some of the units were wrong

WimpSim v1 -> v2 Changelogs

Full list is available at: https://confluence.slac.stanford.edu/display/CDMS/WIMPs

Physics Changes

- In V2, recoil calculations are fixed, scattering angles are not all in the same direction, and momentum is conserved
- In V2, the Helm form factor is now calculated correctly
- In V2 the WIMP velocity magnitude is now a Maxwell-Boltzmann with a mean at 220 km/s
- Units are now all fixed and are in agreement with SuperSim units

Code Changes

- Multithreading
- Expanded ROOT output (WimpSim tree) with more information about the momentums, Scattering Angles, WIMP relative velocities
- Code is now a singleton class with methods instead of functions
- Code is now C++ instead of a mixture of C and C++
- Data members are now being initialized with a zero value
- Added comments to add readability
- Version tracking

Recoil Energy Distribution Calculation

$$\mathrm{d}n = n_0 \frac{f(\boldsymbol{v}, \boldsymbol{v}_E)}{k} \mathrm{d}^3\boldsymbol{v}$$

$$\mathrm{d}R = N_T v \, \mathrm{d}\sigma \, \mathrm{d}n = N_T n_0 \, \mathrm{d}\sigma \frac{v f(\boldsymbol{v}, \boldsymbol{v}_E)}{k} \mathrm{d}^3\boldsymbol{v}$$

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{r}} = n_0 N_T \int_0^{v_\mathrm{max}} \frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{r}} \frac{v f(\boldsymbol{v}, \boldsymbol{v}_E)}{k} \mathrm{d}^3\boldsymbol{v}$$

$$E_\mathrm{r} = \frac{\mu_T^2 v^2}{m_T}(1 - \cos\theta) \qquad v_\mathrm{min} = \sqrt{\frac{m_T E_\mathrm{r}}{2\mu_T^2}}$$

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{r}} = n_0 N_T \int_{v_\mathrm{min}}^{v_\mathrm{max}} \frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{r}} \frac{v f(\boldsymbol{v}, \boldsymbol{v}_E)}{k} \mathrm{d}^3\boldsymbol{v}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{r}} = \frac{\mathrm{d}\sigma_\mathrm{SI}}{\mathrm{d}E_\mathrm{r}} + \frac{\mathrm{d}\sigma_\mathrm{SD}}{\mathrm{d}E_\mathrm{r}} \qquad \left.\frac{\mathrm{d}\sigma_\mathrm{SI}}{\mathrm{d}q^2}\right|_{q\to 0} = \frac{1}{\pi v^2}\left[f_p Z + (A - Z) f_n\right]^2$$

$$\frac{\mathrm{d}\sigma_\mathrm{SI}}{\mathrm{d}q^2} = \frac{1}{\pi v^2}\left[f_p Z + (A - Z) f_n\right]^2 F_\mathrm{SI}^2(q)$$

$$\sigma_0^\mathrm{SI} = \int_0^{4\mu_T^2 v^2} \left.\frac{\mathrm{d}\sigma_\mathrm{SI}}{\mathrm{d}q^2}\right|_{q\to 0} \mathrm{d}q^2 = \frac{4\mu_T^2}{\pi}\left[f_p Z + (A - Z) f_n\right]^2$$

$$\frac{\mathrm{d}\sigma_\mathrm{SI}}{\mathrm{d}E_\mathrm{r}} = \frac{m_T}{2\mu_T^2 v^2}\sigma_0^\mathrm{SI} F_\mathrm{SI}^2(q)$$

dn	Differential number density of WIMPs
n_0	WIMP number density
$f(v,v_E)$	Velocity distribution function of the Dark Matter halo, with respect to the Earth
v	WIMP velocity
v_E	Earth velocity
N_T	Nuclei per unit mass
dσ	Differential WIMP-nucleus interaction cross section
dR	Differential event rate per mass
θ	Recoiling angle in the center of mass frame
m_T	Nucleus mass
μ_T	Reduced mass of the WIMP-nucleus system
f_p	Proton coupling
f_n	Neutron coupling
A	Mass number
Z	Atomic number

Helm form factor

$$\frac{d\sigma_T}{dQ} = \frac{m_T}{2\mu_T^2 v^2}\left[\sigma_T^{SI} F_{SI}^2(Q) + \sigma_T^{SD} F_{SD}^2(Q)\right]$$

$$F_{SI}(q) = 3\frac{j_1(qr_n)}{qr_n} e^{-\frac{1}{2}(qs)^2}$$
$$= 3\frac{\sin(qr_n) - qr_n\cos(qr_n)}{(qr_n)^3} e^{-\frac{1}{2}(qs)^2}$$

$$\sigma_T^{SI} = \frac{4}{\pi}\mu_T^2\left[Zf_p + (A-Z)f_n\right]^2$$

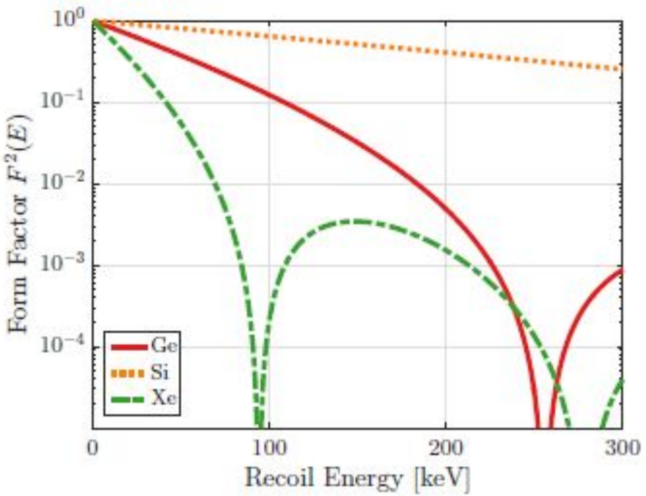
$$r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$$

$$c \simeq 1.23A^{1/3} - 0.60 \text{ fm}$$

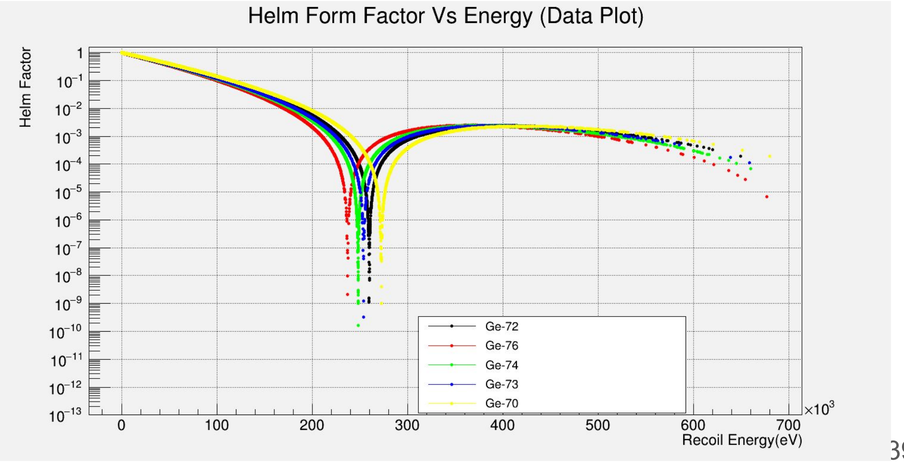
$$s \simeq 0.9 \text{ fm}$$

$$a \simeq 0.52 \text{ fm}$$

A is Atomic Mass

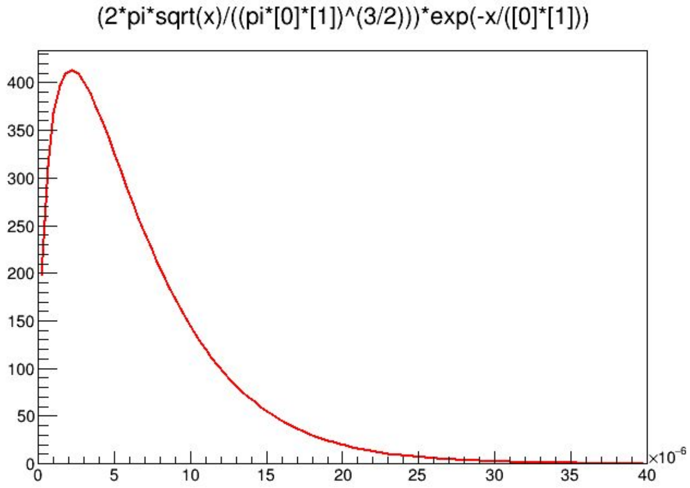


[1]



Thermal Motion

- The detectors are working at 50mK
- The mean kinetic energy of nucleons would be around 6.45 nano electron Volt
- Mass of Germanium atom is around 67 GeV/c^2
- The mean momentum is sqrt(2* 4.6*10^{-9} * 67*10^9)=24.83 eV/c. The mean velocity is (24.83/67*10^9) = 0.37*10^{-9} c = 11.1cm/s
- WIMP velocities are around 220 m/s or around 0.7*10^{-6}c, so for a mass of 5 GeV/c^2 WIMP the momentum is 0.7*10^{-6} * 5*10^9= 3.5*10^3 eV/c
- This means that the momentum of a WIMP is just under 200 times more momentum than a Germanium atom
- To a good degree of approximation the Germanium atom is at rest for our calculations



the (unnormalized) Maxwell-Boltzman distribution plot at T=50mK. X axis is neV (ignoring the 10^-6 as I produced meV plot at first)

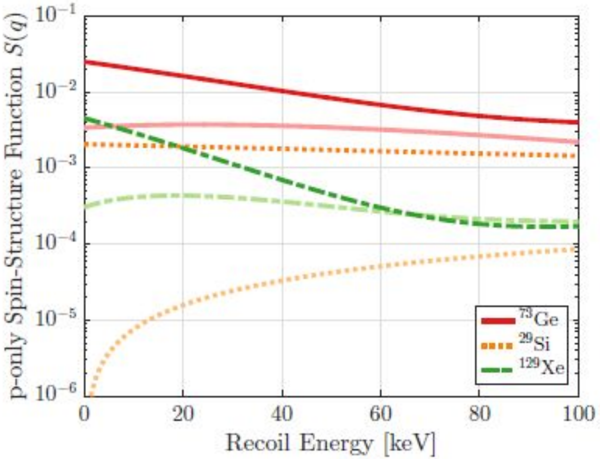
https://en.wikipedia.org/wiki/Boltzmann_constant

K = 8.6E-5 eV/K

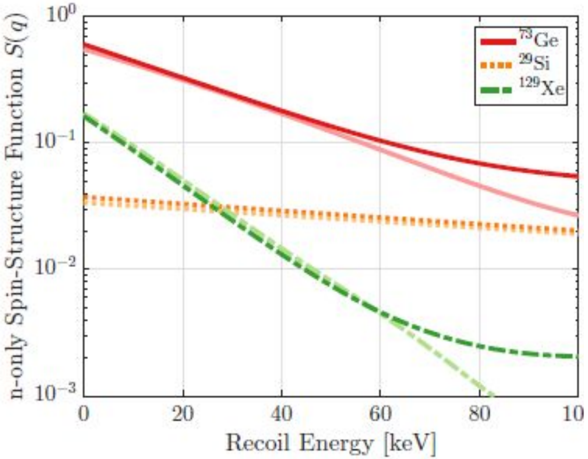
MeanE = 3/2*KT = 1.5*50*10^{-6}K * 8.6*10^{-5} = 6.45*10^{-9} eV

Spin Dependent Form Factor

- The SD interaction does not have the coherence advantage of the SI case. Nucleons with opposite spin interfere destructively meaning that the interaction is dominated by unpaired nucleons.
- Nuclear structure has large effects on the zero momentum limit cross sections
- Results on the standard cross section are normalized to that of a free nucleon, either a proton or neutron



(a) Proton-Only Coupling



(b) Neutron-Only Coupling

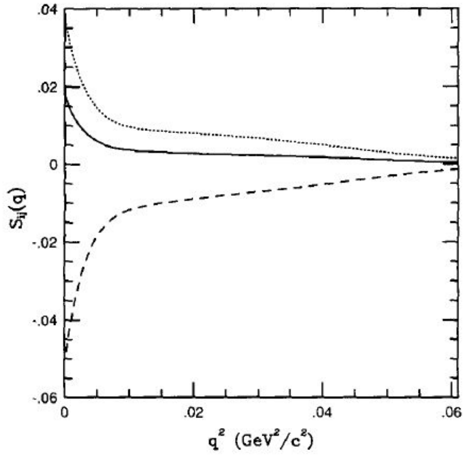


Fig. 3. The partial structure functions $S_{00}(q)$ (dotted line), $S_{11}(q)$ (solid line), and $S_{01}(q)$ (dashed line) versus q^2 in ^{131}Xe. When $a_0 = a_1$ the cross section is small because the protons are mostly in angular-momentum zero pairs.

Spin Dependent Form Factor

$$F^2_{\mathrm{SD}} \equiv S(q)/S(0).$$

$$S_T(q) = a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q)$$

$$a_0 = a_p + a_n$$

$$a_1 = a_p - a_n$$

$$\begin{aligned} S_T(0) &= \frac{(2J+1)(J+1)}{4\pi J} \left|(a_0 + a_1') \langle S_p \rangle + (a_0 - a_1') \langle S_n \rangle\right|^2 \\ &\approx \frac{(2J+1)(J+1)}{\pi J} \left|a_p \langle S_p \rangle + a_n \langle S_n \rangle\right|^2 , \end{aligned}$$

$$a_1' = a_1 (1 + \delta a_1(0))$$

$$\begin{aligned} \sigma_0^{\mathrm{SD}} &= \frac{2}{2J+1} \left(\frac{\mu_T}{\mu_{p/n}}\right)^2 \frac{S_T(0)}{S_{p/n}(0)} \sigma^{\mathrm{SD}}_{p/n} \\ &= \frac{1}{3} \frac{J+1}{J} \left(\frac{\mu_T}{\mu_{p/n}}\right)^2 \frac{\left|(a_0 + a_1') \langle S_p \rangle + (a_0 - a_1') \langle S_n \rangle\right|^2}{a^2_{p/n}} \sigma^{\mathrm{SD}}_{p/n} \end{aligned}$$

J	Total Nuclear Spin
S_{00}	Isoscalar function (determined either by experiment or nuclear modeling)
S_{11}	Isovector function (determined either by experiment or nuclear modeling)
S_{01}	Interference function (determined either by experiment or nuclear modeling)
a_0	Isoscalar coupling
a_1	Isovector coupling
a_p	Proton coupling
a_n	Neutron coupling
$<S_p>$	Spin expectation value for proton
$<S_n>$	Spin expectation value for neutron
μ_T	Reduced mass of the WIMP-nucleus system
μ_p	Reduced mass of the WIMP-proton system
μ_n	Reduced mass of the WIMP-neutron system

Calculating the Rate Plot

$$\frac{dR}{dE_r} = \underbrace{\frac{N_T m_T}{2 m_\chi \mu_T^2}}_{\text{Detector}} \cdot \underbrace{\left[\sigma_0^{SI} F_{SI}^2(E_r) + \sigma_0^{SD} F_{SD}^2(E_r)\right]}_{\text{Particle and Nuclear}} \cdot \underbrace{\rho_0 \int_{v_{min}}^{v_{max}} \frac{1}{k} \frac{f(\boldsymbol{v}, \boldsymbol{v}_E)}{v} d^3\boldsymbol{v}}_{\text{Astro}}$$

$$\sigma_0^{SI} = \left(\frac{\mu_T}{\mu_p} A\right)^2 \sigma_p^{SI},$$

$$\sigma_0^{SI} = \int_0^{4\mu_T^2 v^2} \left.\frac{d\sigma_{SI}}{dq^2}\right|_{q\to 0} dq^2 = \frac{4\mu_T^2}{\pi} \left[f_p Z + (A - Z) f_n\right]^2,$$

Fsd part assumed to be 0 (in both theory plot and my plot)

Sigma P is model independent

NT and mT (not sure on this one) are constant between different WIMP masses in the same detector (

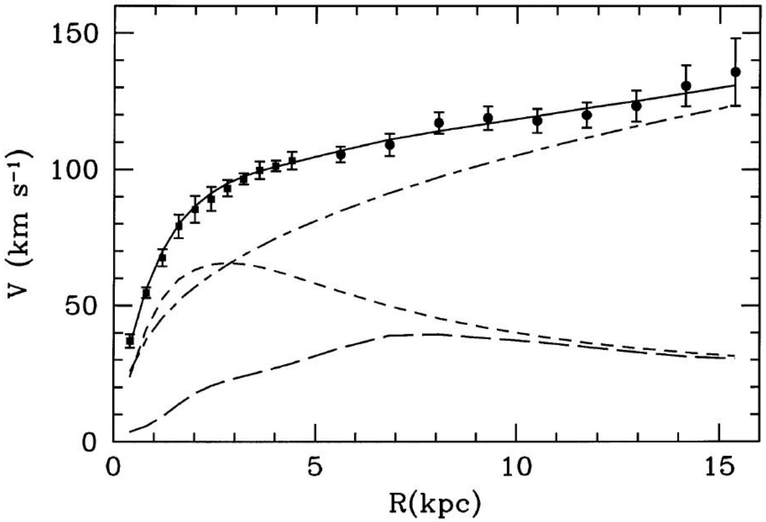
The reduced mass parts cancels each other, so we only need to take into account mX, and Fsi^2 part

So the rate can be simplified to $\dfrac{dR}{dE} = \dfrac{N_T m_T A^2}{2} . \sigma_P^{SI} . \dfrac{1}{\mu_p^2} . F_{SI}^2(E_r). Astro$

What I did was assuming weights are F^2/mp^2*mx instead of only F^2 and then “scale” my plot with a constant amount (C) to match the theory plot for 50 GeV

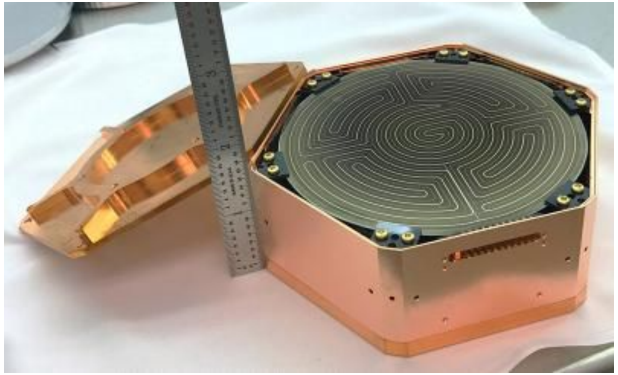
Dark Matter - Evidence

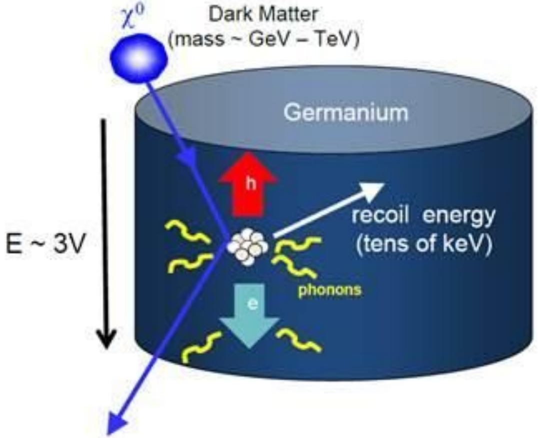
- The figure is M33 rotation curve [10]
  - dot-dashed line is dark matter halo contribution
  - short-dashed line is the stellar disc
  - the longdashed line is H_I gas contribution
- This is suggesting a (roughly) spherical halo of dark matter surrounding the galaxy
- If dark matter halo exists, at high radii the galaxy becomes dark matter halo dominated
- The gas contribution is only H_I (warm ionized gas and molecular gas contribution to the potential well is small)
- This suggests existence of dark matter halo

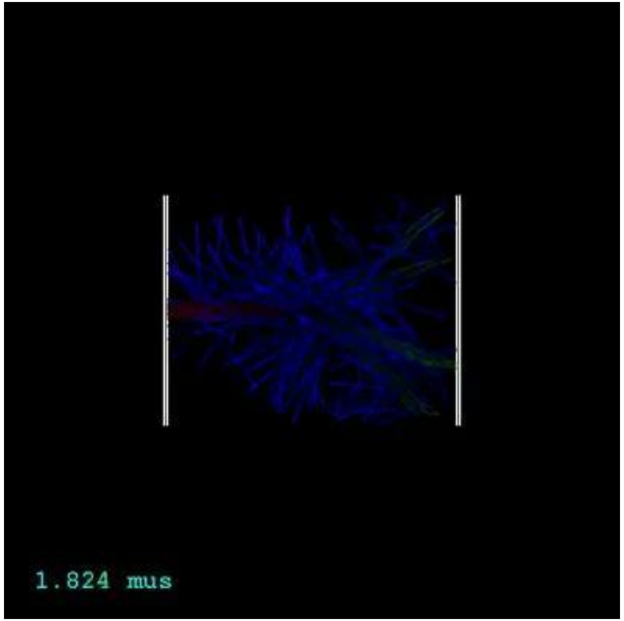


SuperCDMS

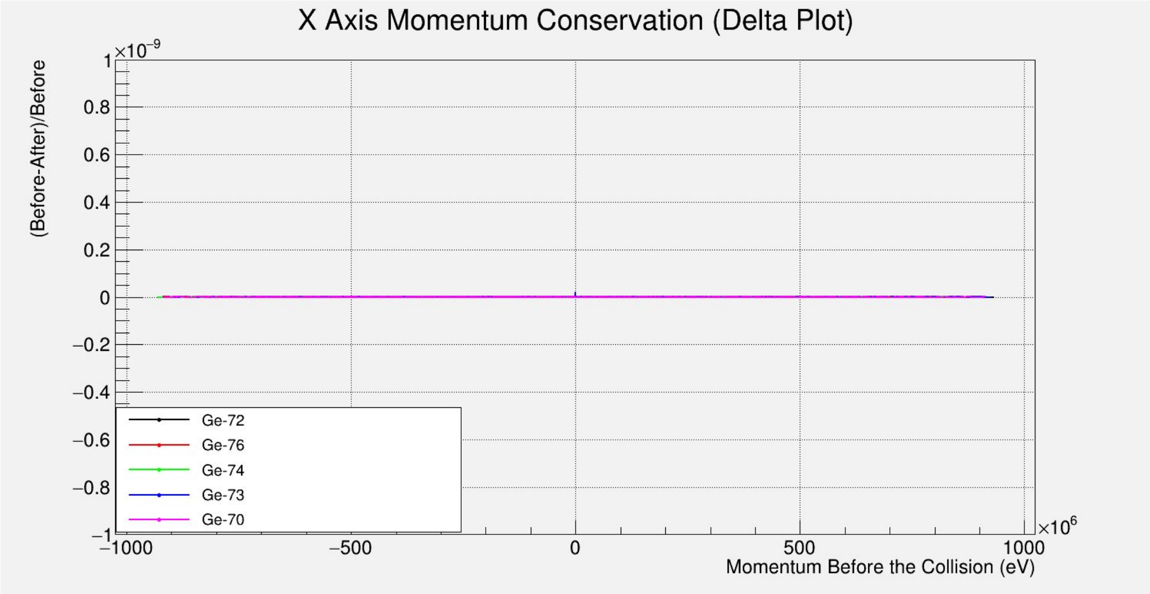
- Incident particles creates conduction band electrons and holes
- Electron and holes gets accelerated in an electric field and radiate phonons
- After energy deposit, phonons move in all directions
- Sensors on top and bottom can absorb phonons to measure energy
- By measuring the the current change we can measure the deposit energy and calculate ***the recoil energy distribution***



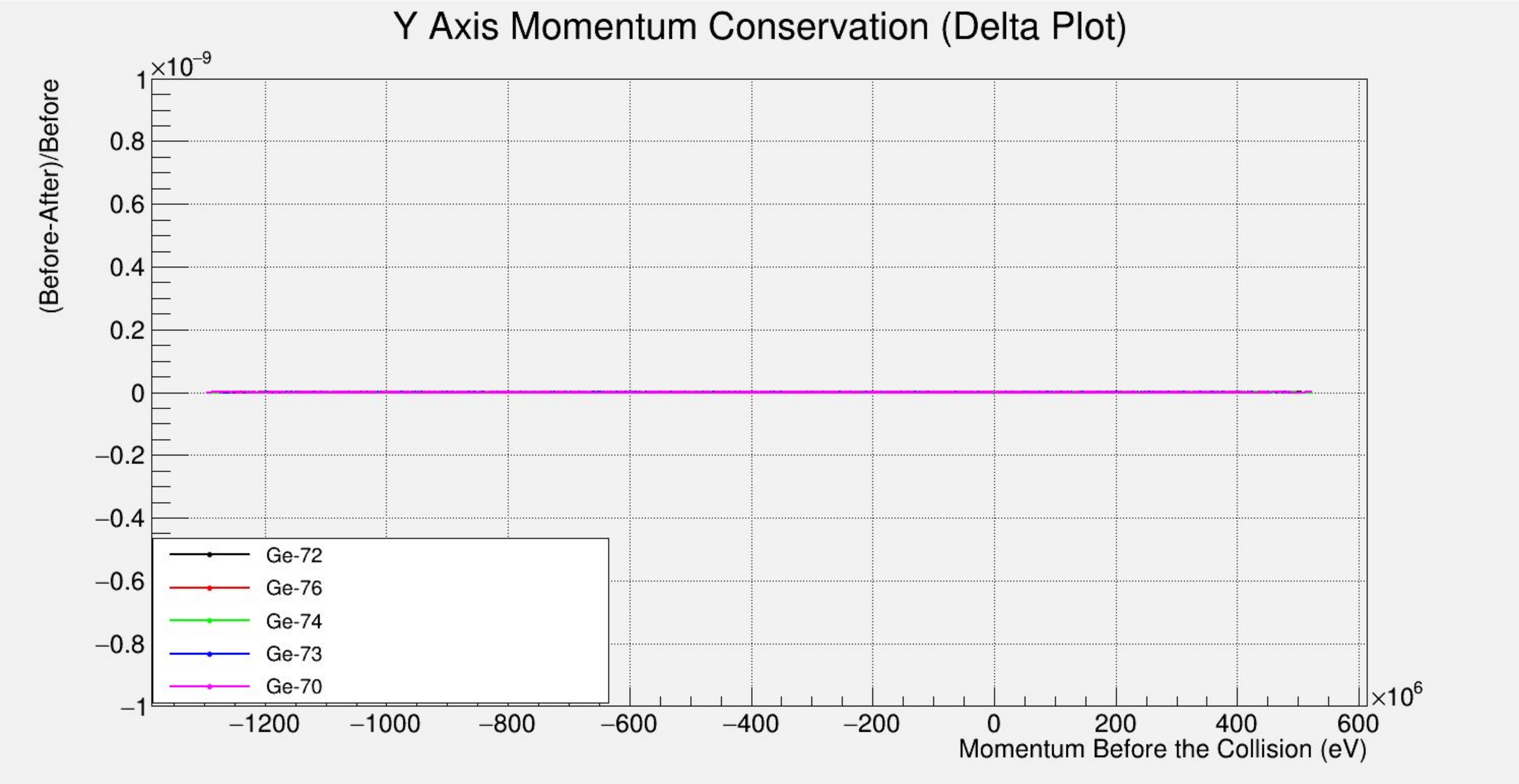




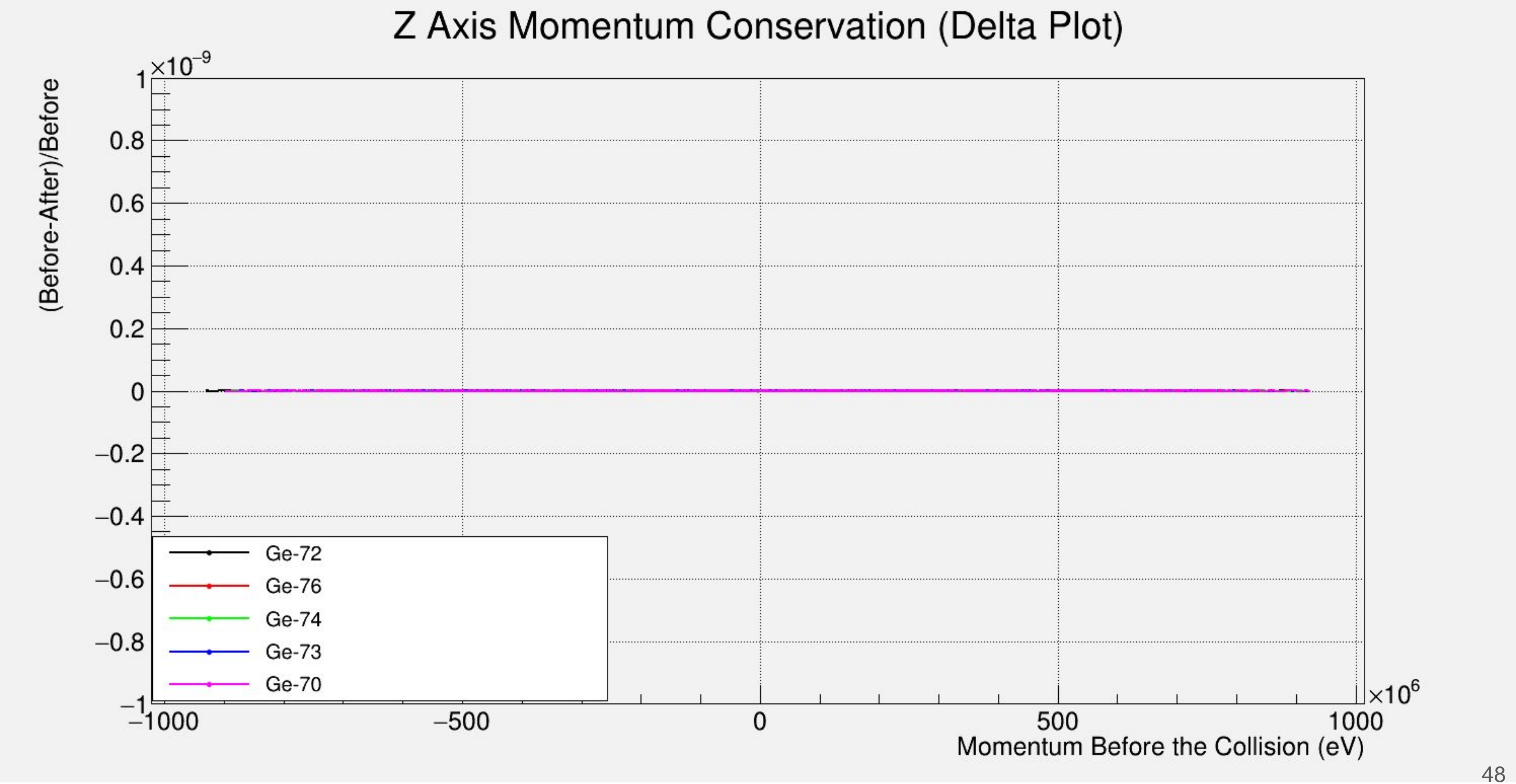
Momentum Conservation



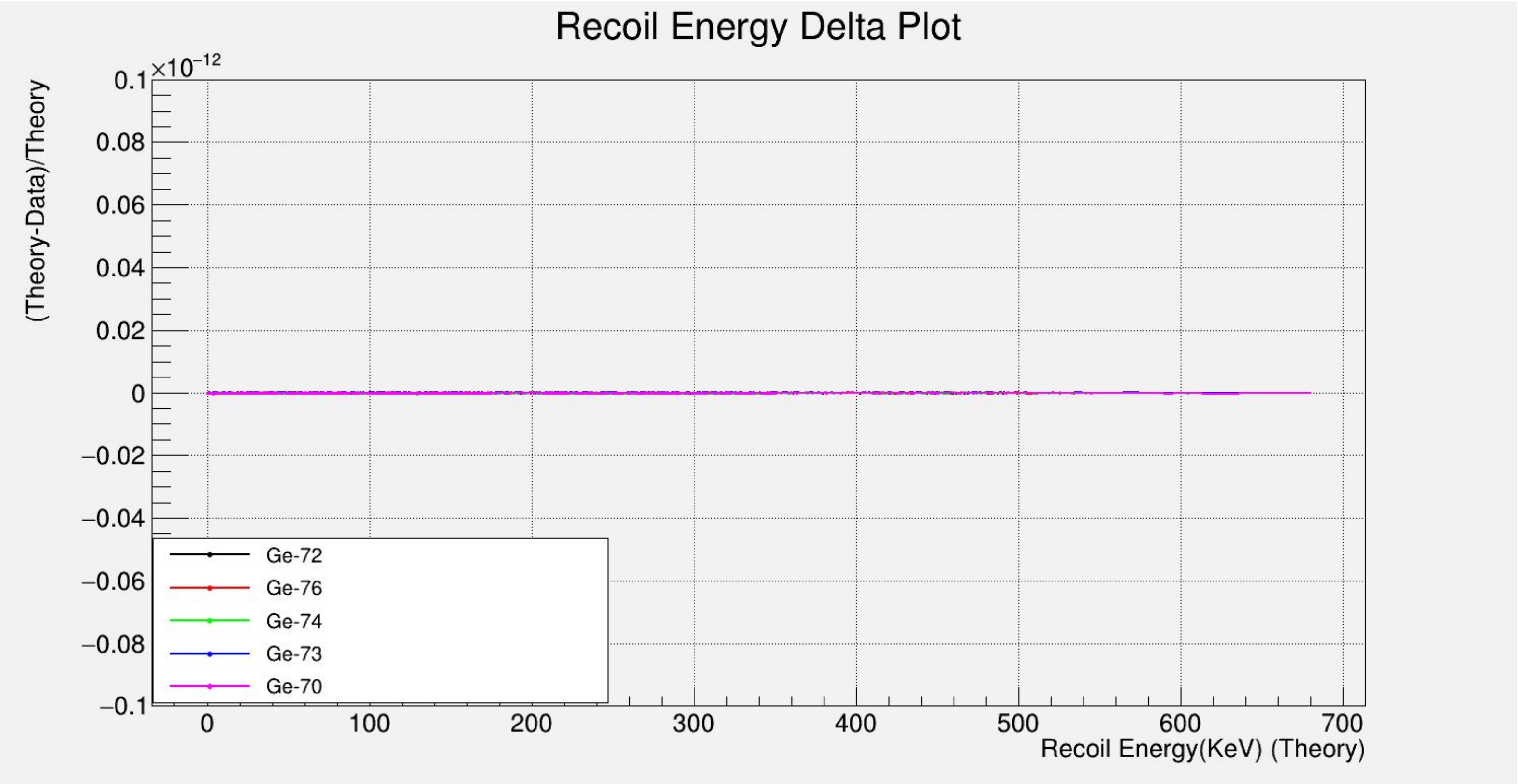
The momentum is conserved in all 3 axis upto 1e^-7 eV (There is systematic error at 1e^-8 due to the 10^16 limit of double variables. Can be solved by using long doubles) Momentum before is WIMP's momentum before the collision, and the Momentum after is WIMP's momentum



Y Axis Momentum Conservation (Delta Plot)
$\times10^{-9}$
(Before-After)/Before
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
Ge-72
Ge-76
Ge-74
Ge-73
Ge-70
−1200
−1000
−800
−600
−400
−200
0
200
400
600
$\times10^{6}$
Momentum Before the Collision (eV)



Z Axis Momentum Conservation (Delta Plot)
(Before-After)/Before
×10^−9
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1000
−500
0
500
1000
×10^6
Momentum Before the Collision (eV)
Ge-72
Ge-76
Ge-74
Ge-73
Ge-70



500 GeV WIMPs

This plot shows that the Recoil Energies distribution results now matches the theory in high mass WIMPs